Prof. Dr. Alfred Toth

Gruppentheoretische Reflexion

1. Wir setzen die Bildung der reflektorischen Relationen der in Toth (2026a) eingeführten Zeichenbezüge fort (vgl. Toth 2026b), indem wir reflektorische Zeichenrelationen und ihre Kompositionsschemata mit Hilfe der gruppentheoretischen Semiotik (vgl. Toth 2009) definieren.

2. Die Zeichenbezüge und ihre reflektorischen Zeichenbezüge

2.1. Die Gruppe (PZ, $\circ_2$)

(PZ, $\circ_2$) wurde bereits von Bogarín (1992) als Gruppe nachgewiesen, nachdem Bense kurz darauf hingewiesen hatte, dass "die kleine semiotische Matrix [...] der Cayleyschen Gruppentafel entspricht" (1986, S. 43).

1. Abgeschlossenheit: $1 \circ_2 1 = 3$; $1 \circ_2 2 = 2 \circ_2 1 = 1$; $1 \circ_2 3 = 3 \circ_2 1 = 2$; $2 \circ_2 2 = 2$; $2 \circ_2 3 = 3 \circ_2 2 = 3$; $3 \circ_2 3 = 1$.

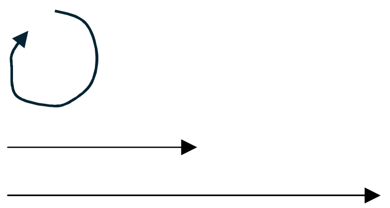
2. Assoziativität: $1 \circ_2 (2 \circ_2 3) = (1 \circ_2 2) \circ_2 3 = 2$; $2 \circ_2 (3 \circ_2 2) = (2 \circ_2 3) \circ_2 2 = 3$, $3 \circ_2 (3 \circ_2 1) = (3 \circ_2 3) \circ_2 1 = 3$, usw.

3. Einselement: $1 \circ_2 2 = 2 \circ_2 1 = 1$; $2 \circ_2 2 = 2$; $3 \circ_2 2 = 2 \circ_2 3 = 3$, d.h. e = 2.

4. Inverses Element: $1^{-1} = 3$, denn $1 \circ_2 3 = 2$; $2^{-1} = 2$ = const., $3^{-1} = 1$, denn $3 \circ_2 1 = 2$.

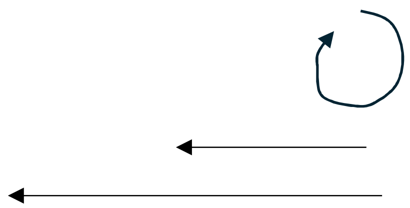
M = (1.1, 1.2, 1.3)

1 2 3



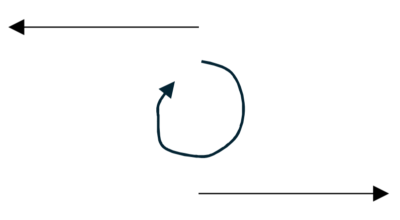
M' = (3.3, 3.2, 3.1)

1 2 3



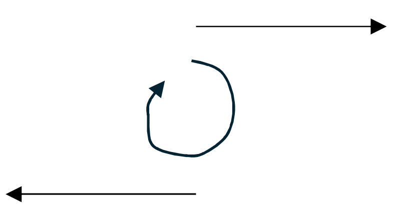
O = (2.1, 2.2, 2.3)

1 2 3



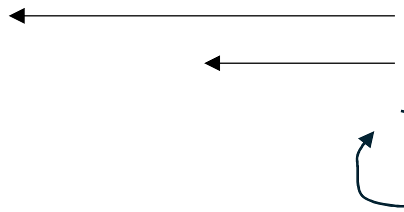
O' = (2.3, 2.2, 2.1)

1 2 3



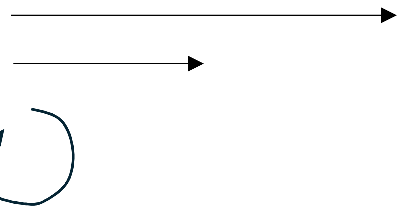
I = (3.1, 3.2, 3.3)

1 2 3



I' = (1.3, 1.2, 1.1)

1 2 3



2.2. Die Gruppe (PZ, $\circ_1$)

1. Abgeschlossenheit: $1 \circ_1 1 = 2$; $1 \circ_1 2 = 2 \circ_1 1 = 3$; $1 \circ_1 3 = 3 \circ_1 1 = 1$; $2 \circ_1 2 = 1$; $2 \circ_1 3 = 3 \circ_1 2 = 2$; $3 \circ_1 3 = 3$.

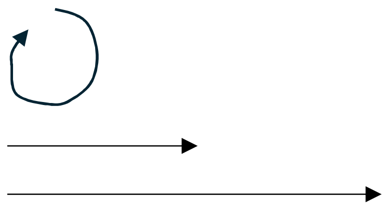
2. Assoziativität: $1 \circ_1 (2 \circ_1 3) = (1 \circ_1 2) \circ_1 3 = 2$; $2 \circ_1 (3 \circ_1 2) = (2 \circ_1 3) \circ_1 2 = 1$, $3 \circ_1 (3 \circ_1 1) = (3 \circ_1 3) \circ_1 1 = 1$, usw.

3. Einselement: $1 \circ_1 3 = 3 \circ_1 1 = 1$; $2 \circ_1 3 = 3 \circ_1 2 = 2$; $3 \circ_1 3 = 3$, d.h. $e = 3$.

4. Inverses Element: $1^{-1} = 2$, denn $1 \circ_1 2 = 3$; $2^{-1} = 1$, denn $2 \circ_1 1 = 3$; $3^{-1} = 3$ = const.

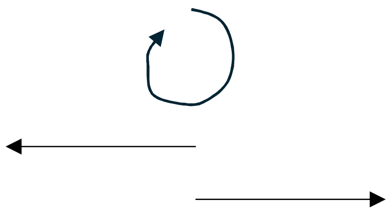
M = (1.1, 1.2, 1.3)

1 2 3



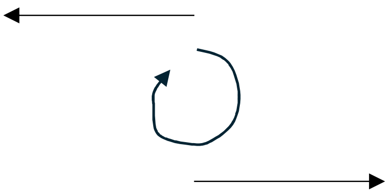
M' = (2.2, 2.1, 2.3)

1 2 3



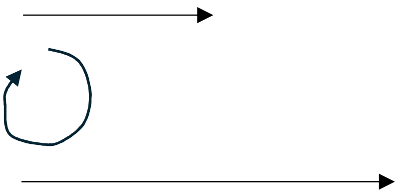
O = (2.1, 2.2, 2.3)

1 2 3



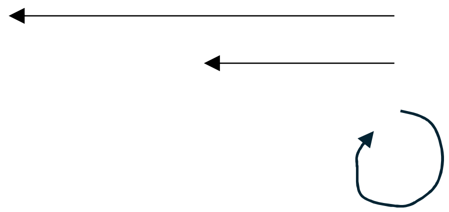
O' = (1.2, 1.1, 1.3)

1 2 3



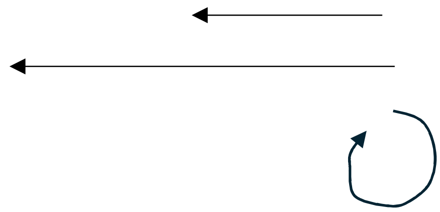
I = (3.1, 3.2, 3.3)

1 2 3



I' = (3.2, 3.1, 3.3)

1 2 3



2.3. Die Gruppe (PZ, $\circ_3$)

1. Abgeschlossenheit: $1 \circ_3 1 = 1$; $1 \circ_3 2 = 2 \circ_3 1 = 2$; $1 \circ_3 3 = 3 \circ_3 1 = 3$; $2 \circ_3 2 = 3$; $2 \circ_3 3 = 3 \circ_3 2 = 1$; $3 \circ_3 3 = 2$.

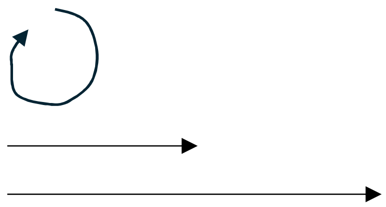
2. Assoziativität: $1 \circ_3 (2 \circ_3 3) = (1 \circ_3 2) \circ_3 3 = 1$; $2 \circ_3 (3 \circ_3 2) = (2 \circ_3 3) \circ_3 2 = 2$, $3 \circ_3 (3 \circ_3 1) = (3 \circ_3 3) \circ_3 1 = 2$, usw.

3. Einselement: $1 \circ_3 1 = 1$; $2 \circ_3 1 = 1 \circ_3 2 = 2$; $3 \circ_3 1 = 1 \circ_3 3 = 3$, d.h. e = 1.

4. Inverses Element: $1^{-1} = 1 =$ const., $2^{-1} = 3$, denn $2 \circ_3 3 = 1$, $3^{-1} = 2$, denn $3 \circ_3 2 = 1$.

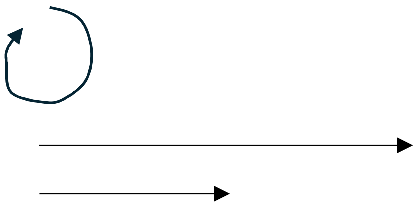
M = (1.1, 1.2, 1.3)

1 2 3



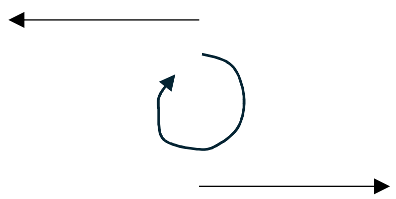
M' = (1.1. 1.3, 1.2)

1 2 3



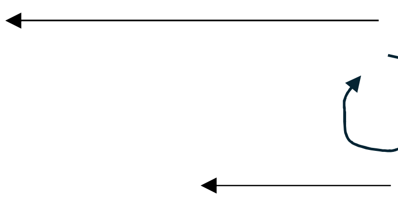
O = (2.1, 2.2, 2.3)

1 2 3

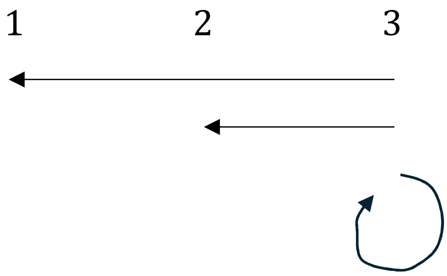


O' = (3.1, 3.3, 3.2)

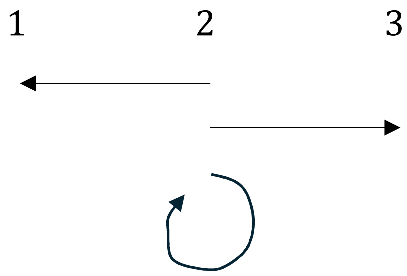
1 2 3



$I = (3.1, 3.2, 3.3)$



$I' = (2.1, 2.3, 2.2)$



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